11-2 Arithmetic Series

Series is the sum of a sequence

8, 12, 16, 20, 24, sequence

8 + 12 + 16 + 20 + 24 series

$S_1 = 8$

$S_2 = 8 + 12 = 20$

$S_3 = 36$

$S_4 = 56$

$S_5 = 80$

In general, $a_1, a_2, a_3, a_4, \ldots a_n$

$S_n = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$

Σ sigma summation sign

8 + 12 + 16 + 20 + 24

Find a_n .

$a_n = 8 + (n-1)4$

$4n - 4$

$a_n = 4n + 4$

$$\sum_{n=1}^{5} (4n + 4)$$

$$\sum_{n=3}^{7} (2n - 3)$$

"the sum of 4n + 4 as n goes from 1 to 5"

Expanded form = $8 + 12 + 16 + 20 + 24$

Put in Expanded form.

$= 3 + 5 + 7 + 9 + 11$

Put the following series into sigma notation:
3 + 6+ 9 + ... + 36

$a_n = a_1 + (n-1)d$
$= 3 + (n-1)3$
$3n - 3$
$a_n = 3n$

$\sum_{n=1}^{12} (3n)$

$36 = 3n$
$12 = n$

Challenge:
Find the sum of the integers from 1 to 100

$1 + 100 = 101$
$2 + 99 = 101$
$3 + 98 = 101$
$\vdots$
$50 + 51 = 101$

5050

$S = \frac{n}{2}(a_1 + a_n)$

Friedrich Gauss

$a_1 =$
$a_{100} =$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Ex:
Find the sum of the given sequence:
5, 10, 15, 20

$S_4 = \frac{4}{2}(5 + 20)$
$= 50$

Ex:
Find the sum of the given sequence:
3, 6, 9, 12, 15, 18

$S_6 = \frac{6}{2}(3 + 18)$
$= 63$

Ex:
Find the sum of the first 50 terms of the given sequence:
3, 6, 9, 12, 15, 18

$a_n = a_1 + (n-1)d$

$S_{50} = \frac{50}{2}(3 + a_{50})$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$\frac{50}{2}[2(3) + 49 \cdot 3]$

$= 3825$

When you don't have the last term, either find it or use this formula.

$$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$

$(a_1 + a_n)$

$a_1 + a_1 + (n-1)d$

Ex:
$a_n = -14$
$n = 9$
$d = -8$
$S_9 =$ _____

$a_n = a_1 + (n-1)d$

$-14 = a_1 + (9-1)(-8)$

$50 = a_1$

$S_9 = \frac{9}{2}(50 + -14)$

$= 162$

HW p586
15-37odd, 40